Hale School

Mathematics Specialist

Test 5 --- Term 3 2018

Applications of Differentiation and Modelling Motion

/ 42

Name: ________________________

Instructions:

- **Calculators are NOT allowed**
- **External notes are not allowed**
- **Duration of test: 45 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

Use exact values in your answers.

Question 1 (3, 3 = 6 marks)

Differentiate the following equations with respect to x .

Please note that you do not need to simplify nor write explicitly in terms of $\frac{dy}{dx}$.

(a) $xy + x^3 = (1+y)^2$

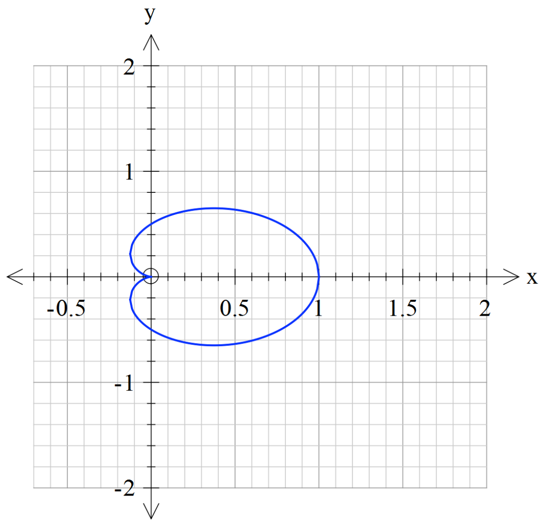
(b) $\frac{1}{\tan y} + x^2 y = \pi$

Question 2 **(5 marks)**

Find the equation of the tangent line to the cardioid curve

$x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at the point $\left(0, \frac{1}{2}\right)$.

(*A cardiod is a heart shaped curve shown below*)



Question 3 **(5 marks)**

A mass has acceleration $a \ m/s^2$ given by $a = v^2 - 3$, where $v \ m/s$ is the velocity of the mass when it has a displacement of $x \ m$ from the origin,

Find v in terms of x given that $v = -2 \ m/s$ where $x = 1 \ m$.

Question 4 **(3, 2, 2 = 7 marks)**

A body moves such that its displacement from some fixed point O at time t seconds is given by $x = 3 + 6\cos \pi t$.

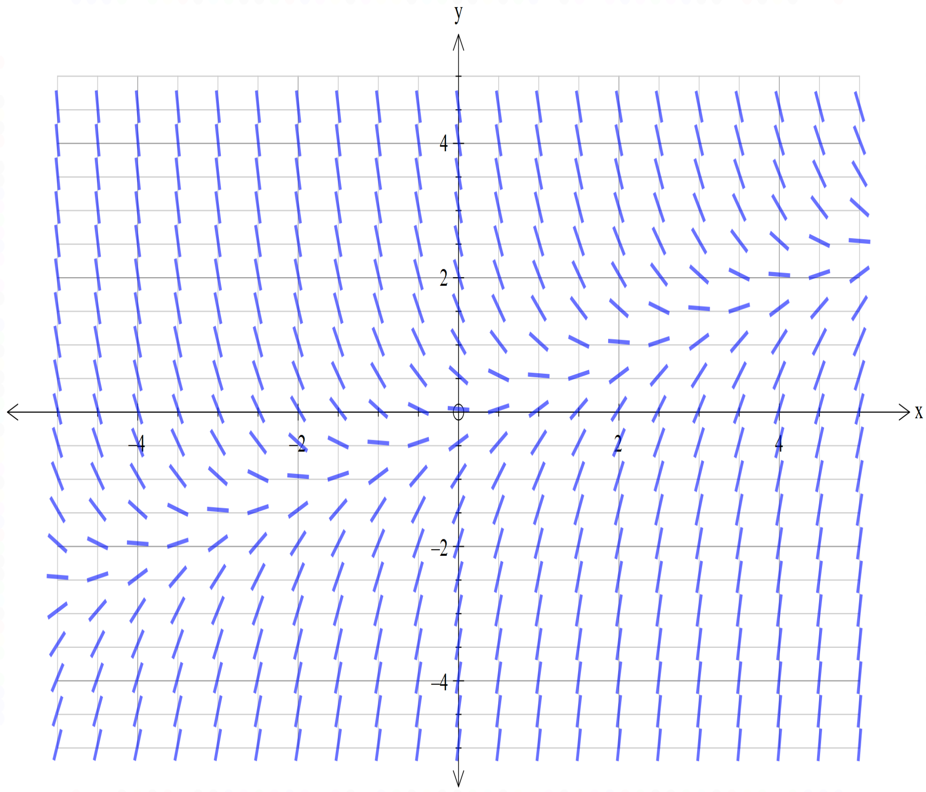
(a) Using the substitution $b = x - 3$, show that the motion is simple harmonic.

(b) What is the period and amplitude of the motion?

(c) Write an expression to determine the distance travelled by the body in the first 10 secs of motion. (You are not required to evaluate.)

Question 5 **(2, 2 = 4 marks)**

A first-order differential equation has a slope field as shown in the diagram below.



(a) On the above slope field, draw in the curve representing the particular solution with initial condition (-1, 1).

(b) Determine with reasoning, which of the following equations best describes the differential equation.

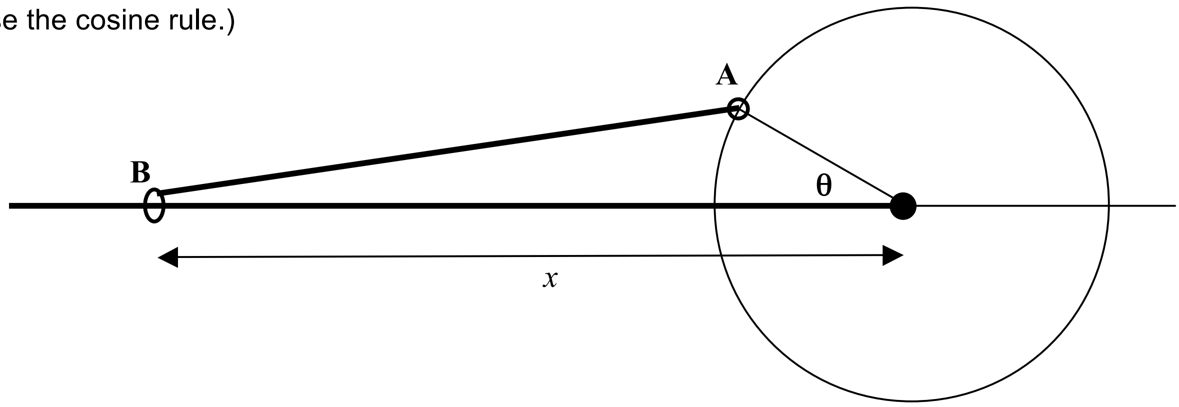
A. $\frac{dy}{dx} = 2x + y$ B. $\frac{dy}{dx} = \frac{y}{2x}$ C. $\frac{dy}{dx} = \frac{2x}{y}$ D. $\frac{dy}{dx} = x - 2y$

Question 6 **(7 marks)**

A rigid rod AB, of length 25 cm, is attached at end A to a circular wheel of radius 7 cm that is turning at 0.5 radians per second. The other end B is attached to a ring that is free to slide along a second horizontal rod.

Determine how fast the ring is moving at the instant when A is in its highest position.

(HINT: Use the cosine rule.)



Question 7 (4, 1, 3 = 8 marks)

When a subject ends, students start to forget the material they have learned. The Ebbinghaus Forgetting Curve assumes that the rate at which a student forgets material is proportional to the difference between the material they currently remember and a positive constant *a*.
Thus, if $y = f(t)$ is the fraction of the original material remembered *t* weeks after a subject has ended, then *y* satisfies the equation:

$$\frac{dy}{dt} = -k(y-a)$$

where k is a positive constant, and a represents the fraction of the original material that will never be forgotten.

(a) Use separation of variables to establish $y = a + Ae^{-kt}$.

(b) Given $y(0) = 1$, find A in terms of a.

(c) Suppose that one week after the final Specialist exam, a student can remember 75% of the material they knew when they sat the exam, and that 25% of the material will never be forgotten. What fraction of the material will they be able to remember 2 weeks after the Specialist exam?